boundary layer. In the case of baffled tanks the above phenomenon was damped to a certain extent due to the controlling importance of turbulence which influences similarly both types of transfer but nevertheless heat transfer remains somewhat larger.

It has already been reported [6] that unbaffled stirred tanks equipped with a large agitator seem to behave like baffled tanks and that, in a reciprocal manner baffled tanks equipped with a small agitator (D/T small) resemble unbaffled tanks, i.e. there is no sharp discontinuity between both systems. The results presented here corroborate the above idea: for example as can be seen on graph 1, j_m for $D/T = 0{\cdot}475$ is larger than for the two other turbines and yet not very different from j_m calculated for baffled tanks.

In order to explain the general differences between the two systems considered above the pumping capacity and its influence on the flow pattern has to be taken into account. For baffled tanks, the pumping capacity is important and the flow leaves the turbine blades like a jet which impinges on the vertical wall of the tank. For unbaffled tanks, the pumping capacity is less important and the fluid velocity is tangential. The difference between these two mixing mechanisms becomes however more or less important according to the size of the turbine. In the case of baffled tanks equipped with a small turbine the liquid jet does not reach the wall due to the drag and the results presented here indicated that the exponent of Reynolds number decreases.

In the case of unbaffled tanks equipped with a large agitator, the importance of the pumping capacity increases particularly when the blade gets closer to the wall; the exponent of Reynolds number is the same as in the case of the baffled tank ($D/T = 0{\cdot}475$). It can be then understood why Mizushina *et al.* [4] have verified the analogy between heat and mass transfer in an unbaffled tank: the tank was indeed equipped with a large agitator ($D/T = 0{\cdot}66$).

4. CONCLUSIONS

It was shown that for baffled tanks the Colburn factors for heat and mass transfer to the wall can be approximated by the same equation. For unbaffled tanks, heat transfer was significantly larger due to the pronounced difference between the values of the Prandtl and the Schmidt numbers and hence to the larger influence of perturbations on the thicker heat-transfer boundary layer. It should be noticed that the exponents of the Prandtl and Schmidt numbers were set equal to 1/3; in fact more sophisticated studies would be required to point out the influence of erratic phenomenon on that exponent [12–13].

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REFERENCES

1. A. Le Lan, Thèse Docteur ès-Sciences, Toulouse (1973).
2. W. S. Askew and R. B. Beckmann, *I/EC Process. Des. Dev.* **4**, 311 (1965).
3. W. S. Askew and R. B. Beckmann, *I/EC Process. Des. Dev.* **5**, 268 (1966).
4. T. Mizushina, R. Ito, S. Hiraoka, A. Ibusuki and I. Sagagushi, *J. Chem. Engng Japan* **2**, 89 (1969).
5. T. H. Chilton, T. B. Drew and R. H. Jebens, *Ind. Engng Chem.* **36**, 510 (1944).
6. A. Le Lan, H. Gibert and H. Angelino, *Chem. Engng Sci.* **27**, 1979 (1972).
7. A. Le Lan and H. Angelino, *Chem. Engng Sci.* **29**, 907 (1974).
8. A. Le Lan and H. Angelino, *Chem. Engng Sci.* **29**, 1557 (1974).
9. A. Le Lan, C. Laguerie and H. Angelino, *Chem. Engng Sci.* To be published.
10. A. P. Colburn, *Trans. Am. Instn Chem. Engrs* **29**, 174 (1933).
11. R. Gardon and J. C. Akfirat, *Int. J. Heat Mass Transfer* **8**, 1261 (1965).
12. V. G. Levich, *Physicochemical Hydrodynamics*. Prentice Hall, Englewood Cliffs (1962).
13. J. T. Davies, *Turbulence Phenomena*. Academic Press, New York (1972).

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LOCAL HEAT-TRANSFER COEFFICIENTS ON THE ROTATING DISK IN STILL AIR

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NOMENCLATURE

A,	area of h-calorimeter front surface [m^2];
B,	relative error of h-calorimeter cooling rate due to errors in temperature measurements [%/deg];
c,	specific heat [J/kg deg];
C,	constant of h-calorimeter, Gc/A [J/m^2deg];
g,	acceleration of gravity [m/s^2];
G,	mass of h-calorimeter [kg];
Gr,	Grashof number, $\beta g(T_w - T_\infty)L^3/\nu^2$;
h,	local heat-transfer coefficient [W/m^2deg];
k,	thermal conductivity [W/m deg];
L,	average equivalent height of h-calorimeter surface as measured from bottom edge of disk [m];

m, relative cooling rate of h-calorimeter, $(\ln\vartheta_1 - \ln\vartheta_2)/\Delta\tau$ [s^{-1}];
Nu, Nusselt number, hR/k;
Pr, Prandtl number;
Q_{loss}, heat loss per time unit through an insulation of the h-calorimeter [W];
R, mean radius of the h-calorimeter [m];
Re, Reynolds number, $\omega R^2/\nu$;
T, temperature [°C].

Greek symbols
β, coefficient of thermal expansion of air [deg^{-1}];
ϑ, temperature differential, $(T - T_\infty)$ [deg];
ν, kinematic viscosity [m^2/s];
ω, angular velocity [s^{-1}];
$\Delta(\ )$, absolute error;
$\Delta\tau$, measurement period [s].

Subscripts
1 and 2, at the beginning and end of the measurement period;
∞, ambient air;
r, errors of random nature;
s, errors of systematic nature but of unknown value;
w, wall surface.

INTRODUCTION

AVERAGE heat transfer on the rotating disk in still air was the most common subject of experimental studies concerning heat transfer in the rotating systems [1, 2]. But not until recently did McComas and Hartnett [3] work out a measurement of local heat-transfer coefficients on the rotating disk in still air. However the results of their investigations about the transient region are rather inadequate.

The aim of this paper is to report a more accurate distribution of the local heat-transfer coefficients over the isothermal disk surface rotating in still air.

EXPERIMENTAL METHOD AND APPARATUS

The measurements of heat-transfer coefficients have been carried out by the ring-shaped h-calorimeter fitted in the

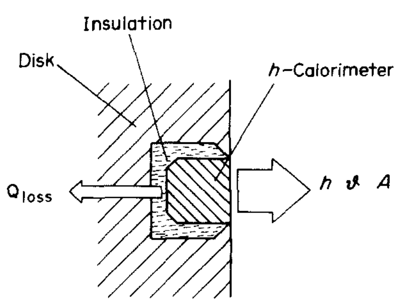

FIG. 1. h-Calorimeter.

surface of the rotating disk (see Fig. 1). The heat balance equation of the h-calorimeter is as follows:

$$-Gc\,\mathrm{d}\vartheta = h\vartheta A\,\mathrm{d}\tau + Q_{loss}\,.\,\mathrm{d}\tau. \tag{1}$$

When the temperatures of the disk and h-calorimeter get sufficiently close to one another, the heat losses through insulation Q_{loss} can be neglected. Then the heat-transfer coefficient is:

$$h = Cm \tag{2}$$

where: $C = Gc/A$—constant of h-calorimeter, and $m = (\ln\vartheta_1 - \ln\vartheta_2)/\Delta\tau$—relative cooling rate of h-calorimeter (see Fig. 2).

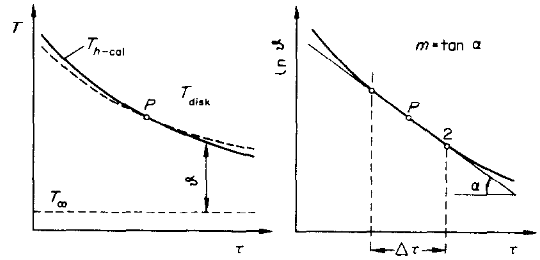

FIG. 2. Relative cooling rate of h-calorimeter.

Error analysis

In order to find out the most convenient conditions of measurement of the relative cooling rate, we will determine its maximum error:

$$\frac{\mathrm{d}m}{m} \triangleq \frac{\Delta m}{m} = \frac{|\Delta(\Delta\tau)|}{\Delta\tau} + \underbrace{\frac{1}{\ln(\vartheta_1/\vartheta_2)}\left(\frac{1}{\vartheta_1} + \frac{1}{\vartheta_2}\right)}_{B_r}|\Delta T_r| + \underbrace{\frac{1}{\ln(\vartheta_1/\vartheta_2)}\left(\frac{1}{\vartheta_1} - \frac{1}{\vartheta_2}\right)}_{B_s}|\Delta T_s|. \tag{3}$$

Figure 3 illustrates the curves of B_r and B_s values, depending on the measurement conditions. It can be observed, that a

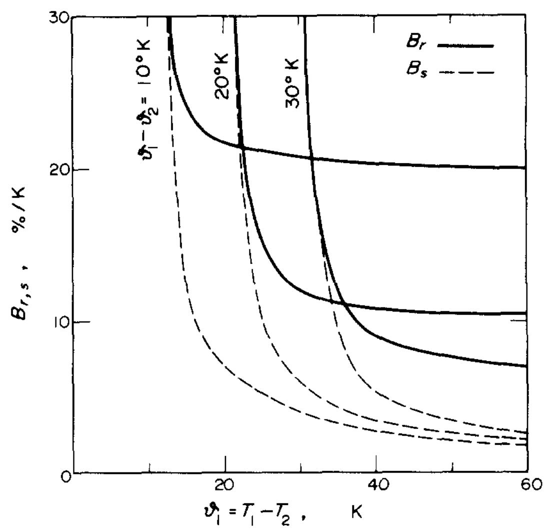

FIG. 3. Plot of the B_r and B_s values as a function of temperature at the beginning of the measurement period and a temperature difference in this period as a parameter.

random nature error in temperature measurement ΔT_r has a greater effect on the error of m-measurement then an error of a systematic nature ΔT_s (i.e. error remaining constant throughout the duration of measurement). For example, with a temperature differential $\vartheta_1 - \vartheta_2 = 10$ deg the temperature of h-calorimeter at the beginning of the measurement period should be $\vartheta_1 = 30$ deg. Then the error of m-value will not exceed:

$$\frac{\Delta m}{m} = B_r|\Delta T_r| + B_s|\Delta T_s| = 20\ 0{\cdot}05 + 4\ 0{\cdot}5 = 3{\cdot}0 \text{ per cent}$$

where $\Delta T_r = 0{\cdot}05$ deg (for 0·1 per cent linearity deviation) and $\Delta T_s = 0{\cdot}5$ deg are magnitudes of errors in the temperature measurements carried out with compensographs of 0·25 per cent accuracy.

Apparatus

The test disk was fitted at the end of a horizontal shaft and driven by an electric motor through a belt transmission (see Fig. 4). On the disk of 480 mm in dia. a ring-shaped h-calorimeter of 185·2 mm radius and 20 mm width was mounted, with a 5 mm gap between them. The gap was filled with glass wool. Both the disk face and h-calorimeter surfaces, made of aluminium were polished to reduce heat radiation effect which was allowed for in the measurements of convection coefficients. Flush with the rotating disk surface a fixed panel was fitted forming somewhat its extension. The disk revolutions were measured by means of a digital speedometer. A set of infrared lamps and a combined fan-heater were used to heat the disk to 120°C. For temperature measurement copper–constantin thermocouples were applied. Impulses from the rotating thermocouple junctions were transmitted to compensograph via copper–graphite brushes. Ambient air temperature was measured in disk centre-line at a distance of 300 mm from the disk surface.

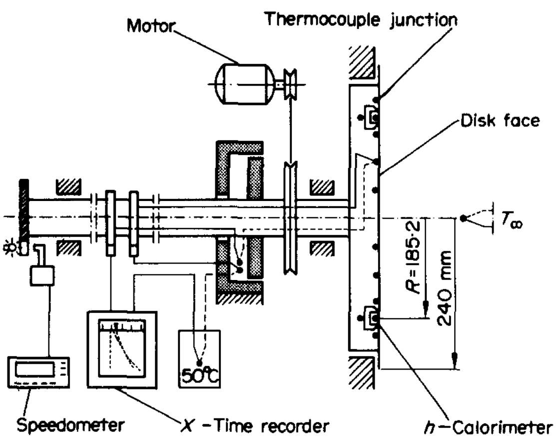

FIG. 4. Schematic diagram of apparatus.

Measurement technique

The test disk and the h-calorimeter were heat to approx. 100°C. Then the h-calorimeter was heated or cooled by some degrees against the disk temperature. Then the disk was cooled down at its given rotating speed. Basing on the recorded changes of disk and h-calorimeter temperatures the measurement period was determined, in which the intersection of the mean temperatures of disk and h-calorimeter were contained. Levelling of those two temperatures provided the expected disappearance of heat losses and formed the model of the rotating isothermal disk. The heat-transfer coefficient could therefore be calculated from equation (2). On the rotating disk with the laminar boundary layer the disk surface was maintained almost isothermal, with the transient boundary layer it was isothermal within $\pm 1{\cdot}5$ per cent and with the turbulent boundary layer it was isothermal within ± 3 per cent. For the duration of measurement periods the temperature of disk was reduced from approx. 80 to 70°C at ambient temperature of about 23°C.

RESULTS

Figure 5 presents results of measurements of the local heat-transfer coefficients on the rotating isothermal disk in still air. Values of the local Nusselt number against the rotational Reynolds number clearly indicate the existence of three regions: laminar, transition, turbulent.

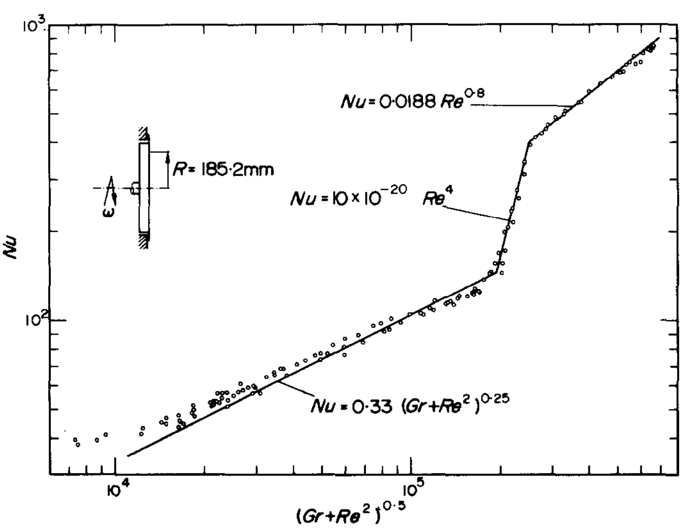

FIG. 5. Local heat transfer from a rotating disk in still air.

The results of measurements within the bracket of Reynolds number from 10^4 to 195000 can be expressed by equation

$$Nu = 0{\cdot}33\,(Gr+Re^2)^{0{\cdot}25}. \quad (4)$$

The effect of free convection was allowed for by Grashof number (as in [4]), the influence of which on the value of heat-transfer coefficient at the local Reynolds numbers greater than 3×10^4 did not exceed 6 per cent. The small deviation of the measurement results from the theoretical values for low Reynolds numbers, may have resulted from a some difference between a theoretical and an experimental flow models due to a final diameter of the test disk. The table below contains a comparison of the results of some experiments with an analitycal prediction. The results of measurements by Cobb and Saunders [5], as well as those by Richardson and Saunders [4], obtained on the same stand are probably too high due to disturbing influence of the shaft and stand elements.

Table 1. Comparison of experimental results with a theoretical solution (for $Pr = 0{\cdot}71$)

Author	$\dfrac{Nu}{(Gr+Re^2)^{0\cdot25}f(Pr=0\cdot71)}$
Experiments of:	
Richardson and Saunders [4]	0·40
Cobb and Saunders [5]	0·36
Kreith, Taylor and Chong [6]	0·34
McComas and Hartnett [3]	0·33
Authors of this paper	0·33
Analytical solution of:	
Hartnett [7]	0·33

Within the range of Reynolds number from 195000 to 250000 there is a fast increase of local heat-transfer coefficients. The limits of this region overlap quite well with the points of stability loss of the laminar boundary layer and the beginning of turbulent boundary layer on the rotating disk, as determined by Gregory and Walker [8].

The local heat transfer in the transition region can be expressed by following equation*

$$Nu = 10 \times 10^{-20}\,Re^4. \quad (5)$$

The measurement results worked out within the range of Reynolds numbers from 250000 to 670000 at the constant radius $R = 185$ mm can be expressed by the equation*

$$Nu = 0{\cdot}0188\,Re^{0{\cdot}8}. \quad (6)$$

An average Nusselt number $\overline{Nu} = Nu/1{\cdot}3 = 0{\cdot}0145\,Re^{0{\cdot}8}$ is 3·7 per cent lower than those obtained by Cobb and Saunders [5].

REFERENCES

1. F. Kreith, Convection heat transfer in rotating systems, in *Advances in Heat Transfer*, Vol. 5, pp. 129–251. Academic Press, New York (1968).
2. L. A. Dorfman, Heat and mass transfer near rotating surfaces (Review), *J. Engng Phys.* **22**(2), 350–362 (1972).
3. S. T. McComas and J. P. Hartnett, Temperature profiles and heat transfer associated with a single disk rotating in still air, in *Proc. Fourth Int Heat Transfer Conference*, Vol. 3, FC 7.7. Versailles (1970).
4. P. D. Richardson and O. A. Saunders, Studies of flow and heat transfer associated with a rotating disk, *J. Mech. Engng Sci.* **5**(4), 336–342 (1963).
5. E. C. Cobb and O. A. Saunders, Heat transfer from a rotating disk, *Proc R. Soc.* **A236**(1206), 343–351 (1956).
6. F. Kreith, J. H. Taylor and J. P. Chong, Heat and mass transfer from a rotating disk, *J. Heat Transfer* **81**, 95–105 (1959).
7. J. P. Hartnett, Heat transfer from a nonisothermal disk rotating in still air, *J. Appl. Mech.* **26**, 672–673 (1959).
8. N. Gregory, J. T. Stuart and W. S. Walker, On the stability of three-dimensional boundary layers with application to the flow due to a rotating disk, *Phil. Trans. R. Soc.* **A248**(943), 155–199 (1955).

*The effect of the finite width of the h-calorimeter was duly allowed for, i.e. lowering of the measured values by 1·6 per cent in the transition region, and by 0·7 per cent in the turbulent region.

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EXACT SOLUTIONS FOR MULTI-DIMENSIONAL RADIATIVE TRANSFER IN NON-ISOTHERMAL SPHERICAL MEDIA

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NOMENCLATURE

Bu, Bouguer number or optical thickness, $Bu = \alpha_a L$;
I, specific radiation intensity;
J, dimensionless average radiation intensity, $J \equiv M/4\sigma T_r^4$;
L, reference length;
l_Θ, l_Φ, l_r, direction-cosines;
M, space integrated radiation intensity, $M \equiv \int I\, d\Omega$;
Q_θ, Q_Φ, Q_r, normalized radiation heat flux, $Q = q/\sigma T_r^4$;
q_θ, q_Φ, q_r, radiation heat flux in θ, Φ and r-direction;
R, position vector in the global spherical coordinate system;

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